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Challenge problems in the characterization of different types of insight present in the solution of mathematical problems

Problemas reto en la caracterización de diferentes tipos de insight presentes en la solución de problemas matemáticos

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RESUMEN

Este artículo tiene como objetivo mostrar algunos problemas para caracterizar los diferentes tipos de conocimiento que los estudiantes muestran en problemas no rutinarios. Sabiendo que existen diversas clasificaciones para el pensamiento matemático, una de ellas se tendrá en cuenta: aquella en la que algunos autores clasifican el pensamiento matemático en dos categorías: convergente y divergente. En particular, para Guilford el pensamiento convergente se basa en la búsqueda de una respuesta determinada o convencional que se identifica como la única solución posible a un problema; y, por otro lado, el pensamiento divergente se identifica con una situación en la que se toman varios caminos para encontrar la mejor y / o nueva solución al problema. Este documento de investigación evidenció cómo, en los dos tipos de pensamiento matemático -tanto convergente como divergente- se pueden describir diferentes tipos de conocimiento y, regularmente, cuando ocurren, terminan en la resolución exitosa del problema

ABSTRACT:

This paper aims at showing some problems to characterize the different types of insight students show in non-routine problems. Knowing there are diverse classifications for mathematical thought, one of them will be taken into account: the one in which some authors classify mathematical thought into two categories: convergent and divergent. For Guilford (1950) convergent thought is based on the search of a determined or conventional answer which is identified as the only possible solution to a problem, [1]. On the other side, divergent thought is identified with a situation in which various roads are taken to find the best and/or new solution to the problem. This research project evidenced how in two types of mathematical thought, both convergent and divergent, different types of insight can be described, and, regularly, when they occur, they end up in the successful solving of the problem.

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1. Introduction

Currently, one can find a number of papers on mathematical education dedicated to divergent and convergent mathematical thought, from primary school to the University.

On the other side, it is considered that during the process of solving a problem there might be little thought jumps corresponding to the overcoming of the blockage the problem creates when there is not an immediate solution. In general, this happens because it is a characteristic of the human brain that, after studying a problem for a while without finding a solution, the subconscious continues working even if the individual is performing other activities, and that, after a period, that can go from hours to years, the solution suddenly presents itself. This is what is described as insight [2], [3], or the experience of illumination that gives a seemingly unsolvable problem a solution.

Some authors, among them [2], assume that this type of cognitive phenomenon is associated with convergent thought and that it produces a high and sudden activation when associating remote ideas; it also activates the pleasure areas of the brain.
The insight is produced when there is an internal unblockage that gives way to the understanding of what was insistently looked for; according to [4], this creative process has four parts: i) preparation ii) incubation, iii) illumination, and iv) verification.
In [5] Davis and Hersh, describe it as 'insight flashes", an element that gives way to the light, a new understanding of the individual.

On the other side, in the work [6-7] Fauconnier and Turner Conceptual Integration Networks it is posited that conceptual integration, in general, is a cognitive operation of the kind of analogy, recursivity, mental models, and conceptual categorization. The authors present dynamic, flexible, and active cognitive operations that come into play at the moment of thinking; they are called cognitive blending, and they are supposed to be related to the insight phenomenon. These authors describe a mental space input structure and a jump or projection to new mental spaces independent of the former, made by an unexpected combination of operations.

2. Methodology

Given the nature of the cognitive phenomenon that is to be studied, a methodology based on the qualitative paradigm is required. In [8] Fraenkel and Wallen (1996) present five basic characteristics of this type of study:

(1) The problem or issue – in its natural environment and context- is the primary and direct source; the labor of the researcher constitutes the key instrument in the research
(2) (Data collection is more verbal than quantitative
(3) Researchers focus both in results and p rocesses
(4) Data analysis is carried out in an inductive manner
(5) It is interesting to know how the subjects think and what the meaning of the point of interest is

The study was implemented in an elective course in the Teaching of Mathematics program at a university in Bogotá, during two semesters in 2015 and 2016.

This elective course was named Development of mathematical thought through problem solving. In both semesters a systematic follow up of five students was made. In this process, class sessions were video, and audio recorded, there were interviews and narratives that were analysed later on.

The systematicity of these actions facilitated a description that allowed to know the habits, beliefs, attitudes, abilities and capacities in mathematical problem solving. In particular, an observation of students' situations, events and characteristics during the problem-solving process that allowed to make a characterization of the types of insight that can be present and their relation with convergent and divergent thought types.

In summary, the Figure 1 shows the methodology used in this research project

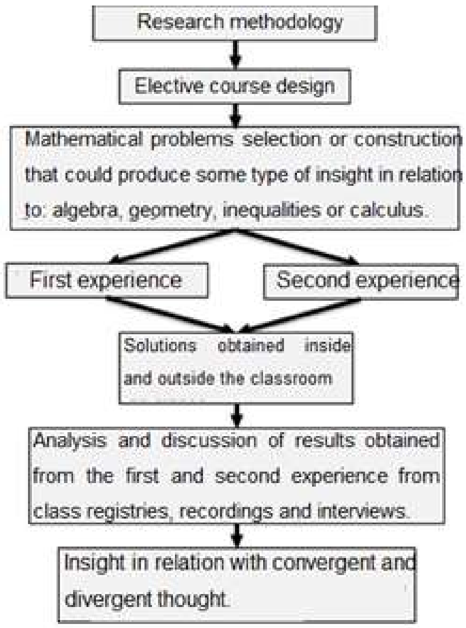

Figure 1. Research methodology [9].

3. Results

From the implementation of the methodology used in this research project the following results are presented. They allowed a characterization of the types of insight present in the solution of the problems.

Problem 1: Determine the area of the shadowed region in the following Figure 2

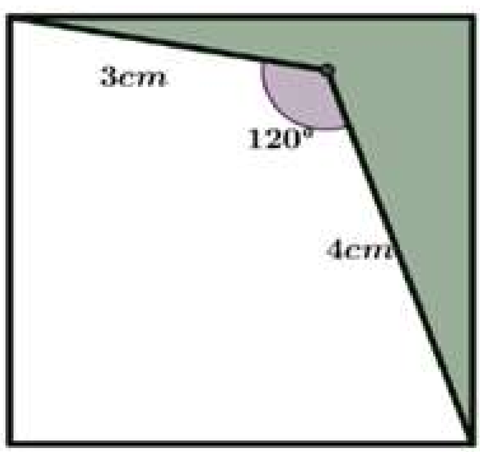

Figure 2. Problem one. Source: own.

Below, we can find the solution of student one, E1, he understands the problem and solves it immediately using a classic result of trigonometry. To do that, he draws the diagonal of the square and names A, B. and C the vortexes of the ABC triangle (Figure 3).

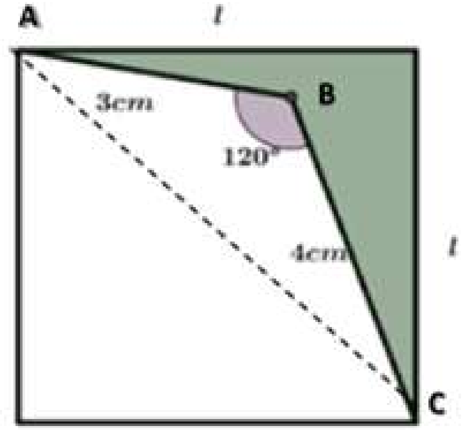

Figure 3. Solving of problem 1 by E1. Source: own.

Then, he calculates the longitude of the diagonal AC using the cosine law $(AC)2 = b2 = (3)2 + (4)2 -(2)(3)$ $cos120° = 25\text{-}12\left(\frac{1}{2}\right)$ $=25+12=37$, where $\sqrt{37}$ After wards he applies the pitagoras theorem, obtains the equality (AC)2 = b2 =2-ℓ2, and thus manages to calculate the total area of the square of side ℓ with with $\ell\,2 = \frac{(AC)^2}{2} = \frac{37}{2}$ As the triangle area formed by the sides of the square and the diagonal is half, then $\frac{37}{4}$ is this value. Following, he calculates the area of the ABC triangle, using the $ABC_\triangle = \frac{1}{2}4 \cdot 3 \cdot sin(120°) = 2 \cdot 3 \cdot sin(120°)$ formula. Finally, the shadowed area is found by subtracting it to the ABC triangle, that is, the shadowed area calculated by E1 was $\frac{37}{4} - 6 \cdot sen(120°) = 9{,}25 - 5.197 = 4.053$ and this was the solution sought for (Figure 4).

Figure 4. E1 original solution to the problem. Source: own

Problem 2. How many squares of different sizes are there in a 1x1, 2x2, 3x3, 8x8 chessboard Could you generalize to a nxn board?

A solution presented by a second student, E2, will be shown. He tries to apply the cosine law, but it doesn`t take him to the solution and a long-time blockage arises when trying to determine the radius of the said circle.
This happened in class. The blockage was surmounted when considering that the incenter of the triangle divides the triangle in three triangles and the radius of this circle is any of the heights corresponding to these three triangles from the incenter and the sum of the area of those three triangles corresponds to the product of the semi perimeter by the radius. It concludes with the correct solution, but with a lapsus: instead of $\pi(0.75)$ he says the solution is $\pi(7.5)$, as we can see in the Figure 6:

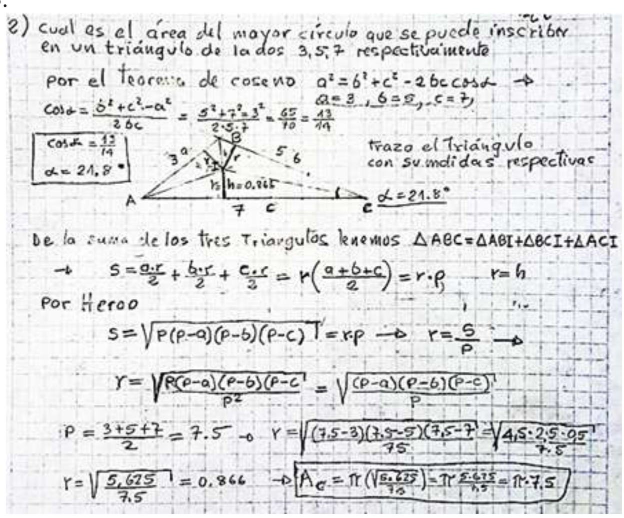

Figure 6. E2 solution to problem 3. Source: own.

From the previous experience inside and outside the classroom, three types of insight could be observed, that, when they happen, they allow to successfully solve a mathematical problem.

(1) Immediate insight. A first approach

The student, in his first attempts, matures some ideas that allow him to solve the mathematical problem inside the classroom; the blockages the student experiences are short term:

(1.1) In general, he understands and tackles the problem in his attempts to come to a solution, with instances where regularities can be seen, invariants that can take to a solution.

(1.2) The student evokes similar experiences with other problems, he relates those experiences to the current problem.

(1.3) An essential factor in the solution of most problems are the pre-concepts the student has on the themes on hand, besides the identification of what is to be determined with the problem.

(1.4) This insight relates to the convergent type of thought in the student, since he comes to a solution almost immediately; his solutions are conventional and he does not require great efforts in his construction.

(2) A posteriori insight. First approach

The student in his first attempts to solve the problem do not advance, the blockage moments, in most cases, can last a considerable amount of time. In general, the student feels motivated to find a solution, besides assuming the problem seriously; new ideas mature, and they can take time. That is what people call incubation. Sometimes, a solution emerges in the least expected moment. But most of the times the solution is obtained with a retro inspection of the work done where students go back and understand better the situation.
An identified factor in students is the different perceptions of the problem itself; since they mature new ideas that are suddenly joined together, and they help come to a solution. There is a relief, joy, and happiness feeling when breaking the frustration of not solving the problem.

(3) Insight by understanding. First approach

The insight by understanding is identified when a solution is found later on; the student generates ideas that allow him to find a new way to solve the problem that can be newer than the first one, which can happen inside or outside of the classroom. This insight is related to the divergent type of thought, in general it has these characteristics:

(3.1) Blockages can be short or long

(3.2) Solutions presented by students are not the expected ones

(3.3) There is a good understanding and tacking of the problem

(3.4) There is novelty in the second solution

From this research project results, there are three kinds of insight in the process of the solution of the problems, inside and outside of the classroom. These insights are related to the convergent and divergent types of thought in the following manner:

Convergent thought is a mental process based on the search of a conventional solution, or related to a problem that does not require a great effort, this solution can be reached with the information available that rests in previous mental spaces, constituted by concepts, definitions, results and previous experiences. That is why the immediate insight is related to this type of thought. Summarizing:

It generally occurs in the classroom

There is a good unerstanding of the problem

We must point out that most of the problems solved inside the classroom, present short-term blockages and in the first attempts to solve them, ideas emerge and develop, and they constitute the solutions of the problems.

The a posteriori insight and the understanding insight are related to the divergent thought in:

The divergent thought is a mental process that must not be necessarily linked to multiple ways of solving a problem; solutions require a great effort and time to develop, besides, it must include fluency, wit and novelty by the part of the student.

We can summarize that:

Blockages last long

There is a good understanding of the problem

Students need short pauses to continue working on the problem

Students try to verify their conjectures rigorously

The problem is assumed seriously and committedly

The solution of problems are presented in a clear and fluid manner, sometimes there are innovations.

Students overcome the blockage of the problem when they inspect back their work and go back; giving way to new ideas that, when developed, allow for the solution of the problem.

4. Conclusions and recommendations

The characteristics that allowed the representation of each type of insight were described. This was possible when looking at the occurrence of these types under a strict observation of the five students during the two semesters.

It is clear that, in the solving process, besides the three types of insight, there are two different levels of insight. It is convenient to look at the types of insight taking into consideration their importance in the field of mathematics, some of them serve as milestones in the history of mathematics.

These two categories of insight must be differentiated. The first is identified as scientific insight and the second as school insight. The scientific is the one that has constructed with its results the building we call a mathematical science. The school insight is the one student experiment within their mathematics classes, and that can guide them to enrich their mathematical thought and to deepen their knowledge in the field, and, in time, to experiment with scientific insights. The three types of insights characterized in this study must be grouped in the school insight. Some scientific insights were mentioned in the introduction.

It is clear that the school insight precedes the scientific; it is quite possible that great mathematicians used the school insight in the construction of their knowledge. There are many examples.

The problems to implement activities of this nature must be designed and/or be very carefully adapted, so that they are interesting enough for students, and so, they can cause short and long-time blockages with their corresponding insight moments inside and outside the classroom.

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